$\mu rad \equiv 10^{-6} \cdot rad$ $mrad \equiv 10^{-3} \cdot rad$ $\mu m \equiv 10^{-6} m$ $nm := 10^{-9} m$

$ms := 10^{-3} \cdot s$ $\mu s := 10^{-6} s$ $ns := 10^{-9} \cdot s$ $kt \equiv 1852 \frac{m}{hr}$ $nmi := 1852 \cdot m$ $℃ := K$

$c := 2.99792458 \cdot 10^{8} \frac{m}{s}$ $h := 6.62606876 \cdot 10^{-34} J \cdot s$ $r_e \equiv 6378140 \cdot m$ $\sigma := 5.670400 \cdot 10^{-8} \cdot \frac{watt}{m^2 \cdot K^4}$

This sheet converts a platform LLA to ECEF, adds to it the vector to a target detected by a sensor on the platform, and converts the target ECEF into LLA.

WGS84
Calculations to be performed as double precision floating point.

This section calculates platform ECEF from LLA. This is a RH coordinate system with positive Z axis pointing north, and positive X axis pointing out from 0 lat, 0 long.

$a := 6378137.0 m$ $f := \frac{1}{298.257223563}$ $b := a \cdot (1 - f)$

$b = 6356.752314245180000\,km$ **WGS84 parameters**

$ecc := \sqrt{\frac{a^2 - b^2}{a^2}}$ $ecc = 8.181919084262150 \times 10^{-2}$

Below are conversion from LLA to ECEF, where la, lo, alt are globablly defined at the bottom of the sheet

$N(la) := \frac{a}{\sqrt{1 - ecc^2 \sin(la)^2}}$ $N(la) = 6384.484879187230000\,km$

$X(la, lo, alt) := (alt + N(la)) \cdot \cos(la) \cdot \cos(lo)$ $X(la, lo, alt) = -653.980490635318000\,km$

$Y(la, lo, alt) := (alt + N(la)) \cdot \cos(la) \cdot \sin(lo)$ $Y(la, lo, alt) = -5313.465452461630000\,km$

$$Z(la, alt) := \left[alt + \left(1 - ecc^2\right) \cdot N(la)\right] \cdot \sin(la) \qquad Z(la, alt) = 3455.568631718600000\,km$$

$$X_{cl1} := X(la, lo, alt) \qquad Y_{cl1} := Y(la, lo, alt) \qquad Z_{cl1} := Z(la, alt)$$

$$X_{cl1} = -653.9805\,km \qquad Y_{cl1} = -5313.4655\,km \qquad Z_{cl1} = 3455.5686\,km$$

$$-652.5565 \qquad -5314.6466 \qquad 3454.0131$$

$$-653.9805 \qquad -5313.4655 \qquad 3455.5686$$

This section rotates the vector to the target heading/uplook to ECEF, and so undoes all the rotations that gets you there in the first place. ECEF is RH with positive Z north. Local level is NED, where positive X points north along the meridian, and positive Z is pointing down. In this example, the platform attitude are ignored, and the sensor only has azimuth and elevation freedom. A complete solution would include platform hdg, pitch, and roll. Therefore, we
> pitch positive look which levels the LOS(x-axis) to NED
> yaw negative hdg to point back to north
> pitch 90 to point the x-axis up and normal to the local level
> pitch positive latitude to level x-axis to the equator (RH rotation means that a positive latitude is a negative pitch rotation angle)
> yaw negative longitude to get to Greenwich Meridian

The vector to the target is now correctly oriented in the ECEF coordinate system and can be added to the platform ECEF coordinates to get the target's ECEF coordinates

$$R_x(\phi) := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \quad R_y(\psi) := \begin{pmatrix} \cos(\psi) & 0 & -\sin(\psi) \\ 0 & 1 & 0 \\ \sin(\psi) & 0 & \cos(\psi) \end{pmatrix} \quad R_z(\theta) := \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Right-handed rotation matrices

$$p(rng) := \begin{pmatrix} rng \\ 0 \\ 0 \end{pmatrix}$$

range vector along the LOS

$$P(la, lo, hdg, down, rng) := R_z(-lo) \cdot R_y(la + 90deg) \cdot R_z(-hdg_{ans}) \cdot R_y(-pit_{ans}) \cdot R_x(-roll_{ans}) \cdot R_z(-hdg) \cdot R_y(-look) \cdot R_x(-roll) \cdot p(rng)$$

$P_t := P(la, lo, hdg, look, rng)$ $\quad P_t = \begin{pmatrix} -1919.2663 \\ 1093.5593 \\ 981.2835 \end{pmatrix} m$

Rotate the range vector into ECEF from heading/uplook in NED space.

$$\sqrt{(P_{t_0})^2 + (P_{t_1})^2 + (P_{t_2})^2} = 2417.1000\,m$$

Adobe Acrobat Document

This is the reference used for both the LLA/ECEF and ECEF/LLA conversion

Closed form solution to convert target ECEF to LLA. This avoids using the usual Helmert's algorithm, which requires at least 2 complete iterations.

$X := X_{cl1} + P_{t_0}$ $\quad Y := Y_{cl1} + P_{t_1}$ $\quad Z := Z_{cl1} + P_{t_2}$

add the target vector to the platform ECEF coordinate

$P(X, Y) := \sqrt{X^2 + Y^2}$ $\quad ep := \sqrt{\frac{a^2 - b^2}{b^2}}$ $\quad ep = 0.0821$ $\quad P(X, Y) = 5352.7096\,km$

$th(X, Y, Z) := atan2(b \cdot P(X, Y), a \cdot Z)$ $\quad th(X, Y, Z) = 32.9404\,deg$

$$La(X, Y, Z) := \operatorname{atan2}\left(P(X, Y) - ecc^2 \cdot a \cdot \cos(th(X, Y, Z))^3, Z + ep^2 \cdot b \cdot \sin(th(X, Y, Z))^3\right)$$

$$La(X, Y, Z) = 0.576451962527726\,\text{rad}$$

$$Lo(X, Y) := \operatorname{atan2}(X, Y)$$

$$Lo(X, Y) = -1.69364109131686\,\text{rad}$$

$$N(la) - \frac{a}{\sqrt{1 - ecc^2 \cdot \sin(La(X, Y, Z))^2}} = -3.96105749253184\,\text{m}$$

$$Alt(X, Y, Z) := \frac{P(X, Y)}{\cos(La(X, Y, Z))} - N(la) \qquad Alt(X, Y, Z) = -64.401492548175200\,\text{m}$$

Global definitions

$la \equiv 0.57624954\,\text{rad}$ la should never be +/-90 deg, since hdg will make no sense. You can make it arbitrarily close to 90, though

$lo \equiv -1.69326026\,\text{rad}$

$alt \equiv 110\,\text{m}$ $rng \equiv 2417.1\,\text{m}$ alt is the platform altitude. The target is presumed to be on the ground, so there is a downlook angle involved

$hdg \equiv 2.3886\,\text{rad}$ $look \equiv 0.5596 \cdot \text{rad}$ $look = 32.0627\,\text{deg}$ "look" is a tricky angle. Local level is NED, so a downlook is a negative angle, and the rotation back to NED results in a positive argument for the pitch rotation.

$roll \equiv -0.5236\,\text{rad}$

$hdg_{ans} \equiv 3\,\text{rad}$ $pit_{ans} \equiv -0.2\,\text{rad}$ $roll_{ans} \equiv 1\,\text{rad}$

$$\sqrt{\left(r_e + alt\right)^2 + \left(rng \cdot \cos(look)\right)^2} - alt - r_e = 0.3289\,m$$

Target alt is due to curvature of the Earth not being accounted for in the look angle calculation, but 3 inches isn't a big deal.